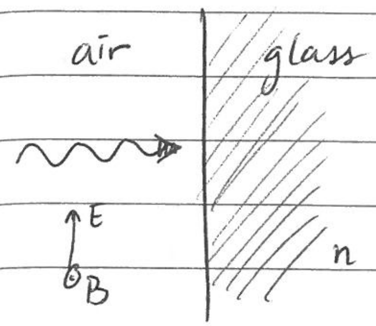
Measuring optical conductivity.



$$B \propto \frac{nE}{c}$$

Remember $\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$ and

slower medium squashes the wave in space.
Bigger curl of $\vec{E}$, bigger B.

$\vec{E}$-field is parallel to any sheet surface charge or effective surface charge.

$$\boxed{E_i + E_r = E_t} \quad \text{—— (1)}$$

There is no surface current $\qquad B_i + B_r = B_t$

$$\boxed{E_i - E_r = n E_t} \quad \text{—— (2)}$$

Because medium changed.

because propogation direction changed

Combine (1) & (2)

$$\boxed{\frac{E_r}{E_i} = \frac{1-n}{1+n}} \quad \& \quad \boxed{\frac{E_t}{E_i} = \frac{2}{1+n}}$$

Also called "local field factor"?

Note that $\left|\frac{E_r}{E_i}\right|^2 + \left|\frac{E_t}{E_i}\right|^2 \neq 1$, need to consider energy flow.

Energy carried by wave is $\propto$ $E \times B$

$$= E \times \frac{nE}{c} = \frac{n}{c} E^2$$

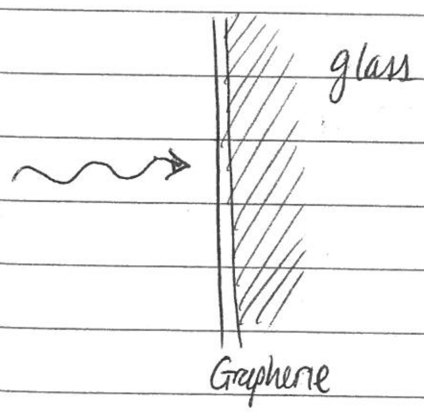
↑ depends on the medium.

So we expect

$$E_i^2 = E_r^2 + nE_t^2 = \frac{(1-n)^2}{(1+n)^2} E_i^2 + \frac{n\,2^2}{(1+n)^2} E_i^2$$

$$= \frac{1-2n+n^2+4n}{(1+n)^2} E_i^2$$

$$= E_i^2$$



$$B_1 - B_2 = \mu_0 j_{2d} = \mu_0 \sigma_{2d} E_t$$

$$B_1 = B_i + B_r = \frac{E_i}{c} - \frac{E_r}{c}$$

$$B_2 = B_t = \frac{nE_t}{c}$$

Dimensional analysis: Relationship between B & E

$[eE] = [evB] = \text{Force}$

$\Rightarrow [B] = \left[\frac{E}{c}\right]$

$$\frac{E_i}{c} - \frac{E_r}{c} - \frac{nE_t}{c} = \mu_0 \sigma_{2d} E_t \qquad \text{— ①}$$

$$E_i + E_r = E_t \qquad \text{— ②}$$

Rearranging eqⁿ. 1 $\quad E_i - E_r = (c\mu_0\sigma + n)E_t$

$$= (n+\delta)E_t$$

where δ is a small ~~correction~~ factor related to the graphene conductivity. (it turns out to be 2.3%) $\delta = Z_0\sigma$

With graphene

$$\frac{E_r}{E_i} = \frac{1-(n+\delta)}{1+(n+\delta)} \qquad \frac{E_t}{E_i} = \frac{2}{1+(n+\delta)}$$

The reflection coefficient $R = \left|\frac{E_r}{E_i}\right|^2$

and transmission coefficient $T = n\left|\frac{E_t}{E_i}\right|^2$

Note that $R+T \neq 1$ because we don't substitute n with $n+\delta$ in the eqn for T.

To make a measurement of δ we need to know R_{gr-s} & R_s

Then $$\frac{R_{gr-s} - R_s}{R_s} = \frac{R_{gr-s}}{R_s} - 1$$

$$= \frac{\left[(1-n)-\delta\right]^2}{\left[(1+n)+\delta\right]^2}\frac{(1+n)^2}{(1-n)^2} - 1$$

$$= \frac{(1-n)^2 - 2\delta(1-n)}{(1+n)^2 + 2\delta(1+n)}\frac{(1+n)^2}{(1-n)^2} - 1$$

$$= \frac{1 - \frac{2\delta}{1-n}}{1 + \frac{2\delta}{1+n}} - 1$$

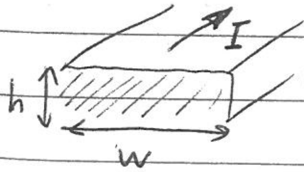
$$= \cancel{1} - \frac{2\delta}{1-n} - \cancel{1} - \frac{2\delta}{1+n}$$

$$= \frac{-2\delta - \cancel{2n\delta} - 2\delta + \cancel{2n\delta}}{1-n^2}$$

$$= \frac{4\delta}{n^2-1} \qquad \text{where } \delta = c\mu_0 \sigma_{2d}$$

Note $\quad c\mu_0 = \frac{\mu_0}{\sqrt{\epsilon_0 \mu_0}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega = Z_0$

To find dimensions of σ_{2d}, note that



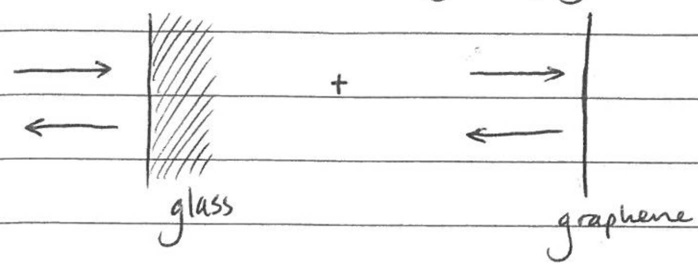
$$G = \frac{\sigma_{3d} \times (\text{height}) \times (\text{width})}{(\text{length})} = \frac{\sigma_{2d} \times (\text{width})}{(\text{length})}$$

units are Ω^{-1}

Theory value for $\sigma_{2d}^{\text{optical}} = \frac{\pi}{4} G_0 = \frac{1}{16{,}500\,\Omega}$

$$\Rightarrow \quad \delta = \frac{377\,\Omega}{16{,}500\,\Omega} = 2.3\%$$

Superposition of glass/graphene reflections



(Approximating the full solution to the boundary value problem)

$$\frac{E_r}{E_i} = \underbrace{\frac{1-n}{1+n}}_{\text{from glass}} - \underbrace{\frac{\sigma Z_0}{2+\sigma Z_0}}_{\text{from graphene}}$$

(i.e. set $n_{substrate} = 1$)

$$\frac{E_r}{E_i} = \frac{(1-n)(2+\sigma Z_0) - \sigma Z_0 (1+n)}{(1+n)(2+\sigma Z_0)}$$

$$= \frac{2 + \cancel{\sigma Z_0} - 2n - n\sigma Z_0 - \cancel{\sigma Z_0} - n\sigma Z_0}{2 + 2\sigma Z_0 + 2n + n\sigma Z_0}$$

$$= \frac{2 - 2n - 2n\sigma Z_0}{2 + 2n + (n+1)\sigma Z_0}$$

$$= \frac{1 - n - n\sigma Z_0}{1 + n + \frac{(n+1)}{2}\sigma Z_0}$$

[Result is an Approximation]

The full solution is $\quad \frac{E_r}{E_i} = \frac{1 - n - \sigma Z_0}{1 + n + \sigma Z_0}$

CONCLUSION: superposition is a reasonable approx. when $n < 2$.

If $n_{glass} = 1.46$ then $\frac{E_r}{E_i} = \frac{1 - 1.46}{1 + 1.46} \approx 0.2$

Compare strength of signal with & without the glass:

W/o Glass

$$\frac{E_r}{E_i} = \frac{-\sigma Z_0}{2 + \sigma Z_0}, \quad I_r \propto |E_r|^2 = \frac{\sigma^2 Z_0^2}{4} |E_i|^2$$

w/ Glass

$$\frac{E_r}{E_i} \cong 0.2 - \frac{\sigma Z_0}{2}, \quad I_r \propto |E_r|^2 = \left(\text{const} - 0.2\sigma Z_0 + \mathcal{O}(\sigma^2)\right)|E_i|^2$$

Ratio of the signals $\frac{0.2\,\sigma Z_0}{\sigma^2 Z_0^2/4} = \frac{0.2 \times 4}{\sigma Z_0}$

$$= \frac{0.8}{0.023} = 35$$

CONCLUSION: Differential reflection from suspended graphene is extremely ~~very~~ small.

Differential reflection from graphene-on-quartz is much bigger because $I_r \propto |E_r|^2$ and the E-fields from quartz refl and graphene reflection are superimposed.